HB^N: A MIM-SECURE HB-LIKE PROTOCOL

Carl Bosley, Stevens Institute of Technology
Joint work with Antonio Nicolosi and Kristiyan Haralambiev

[HB01] and descendents

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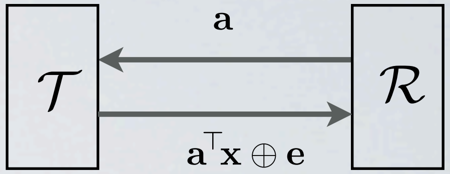

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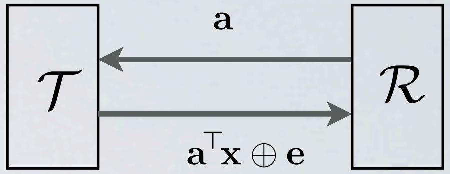

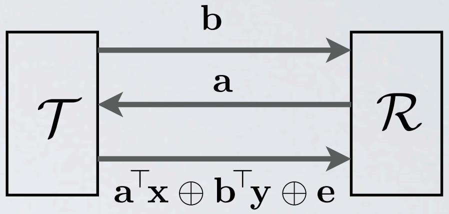
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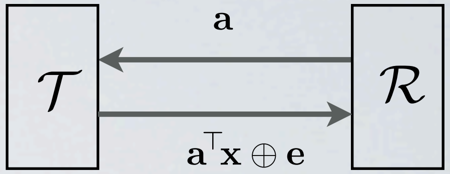

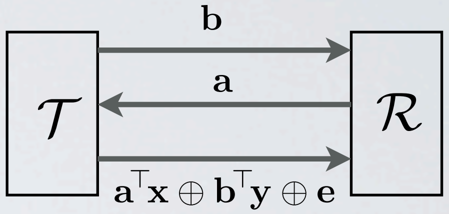
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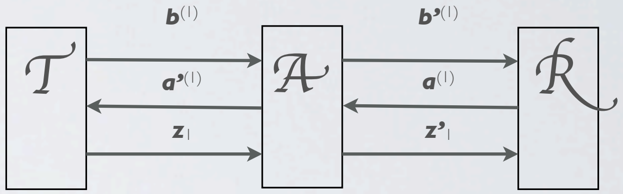

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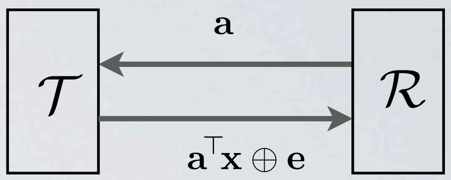

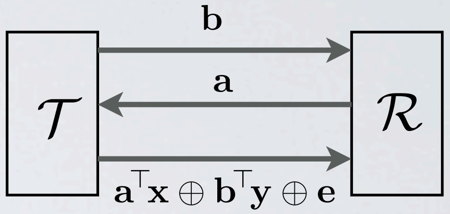

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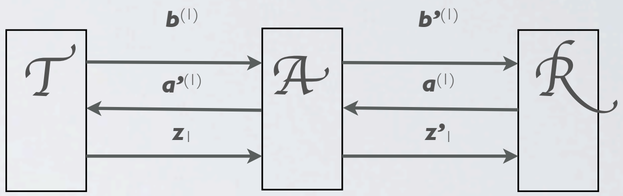

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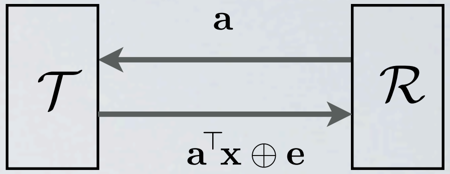

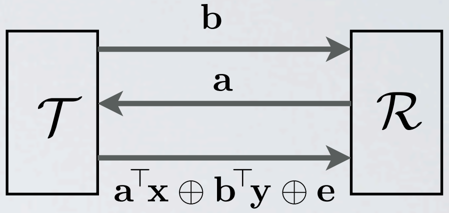
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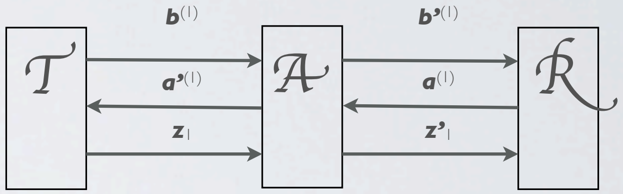

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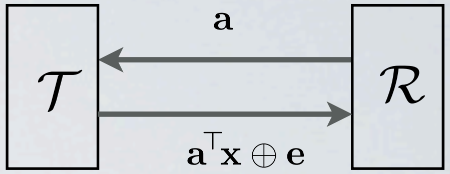

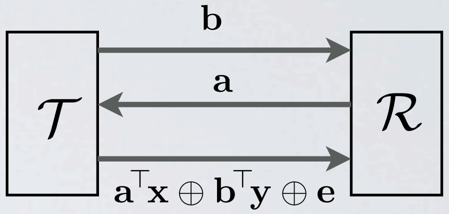

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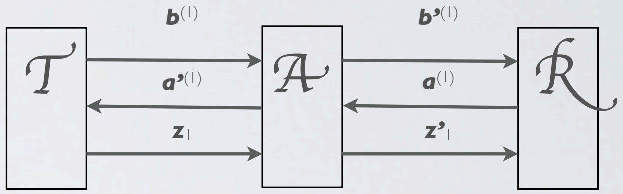

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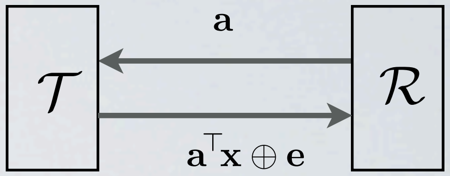

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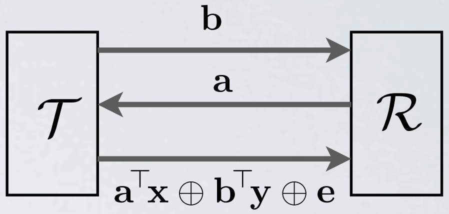
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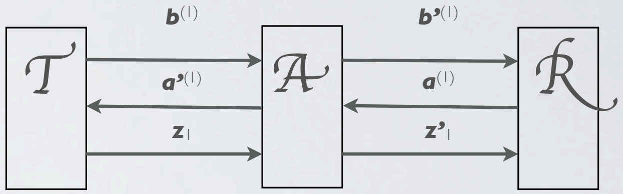

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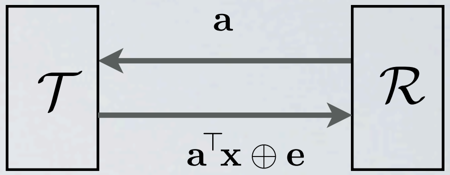

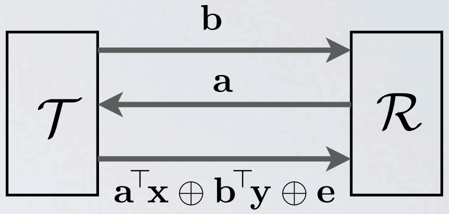
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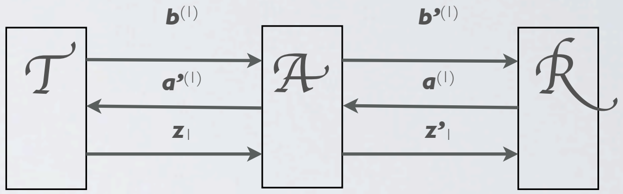

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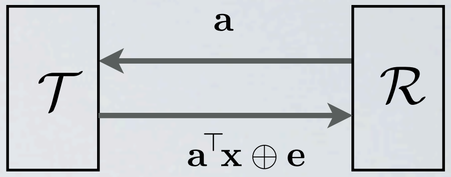

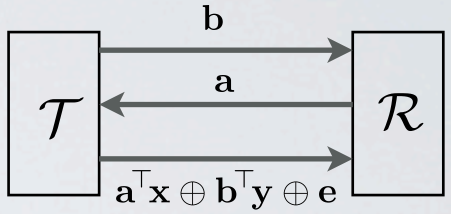
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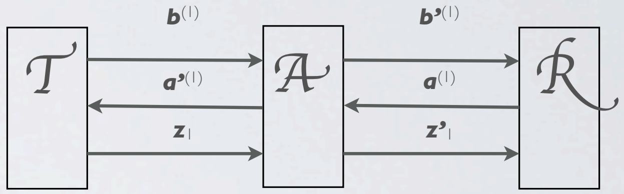

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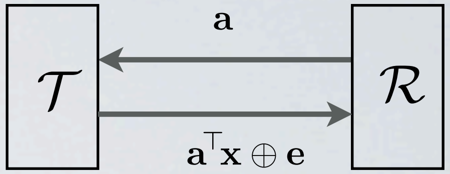

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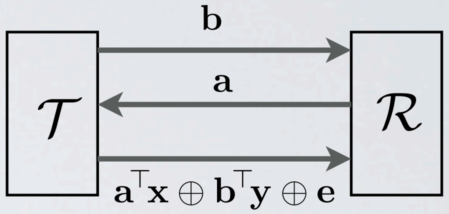

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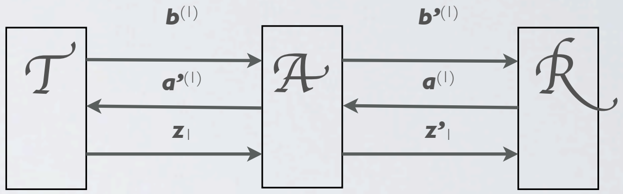

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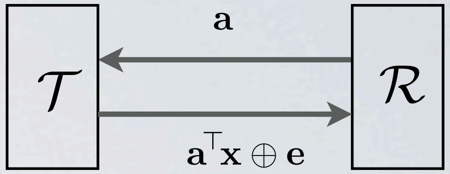

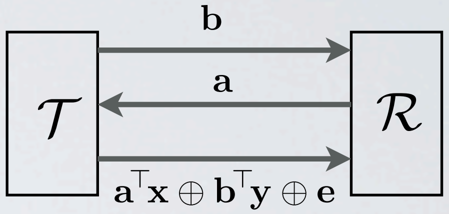
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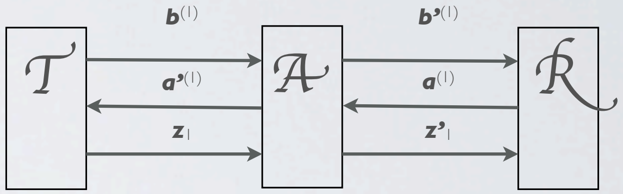

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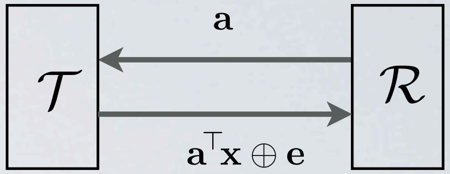

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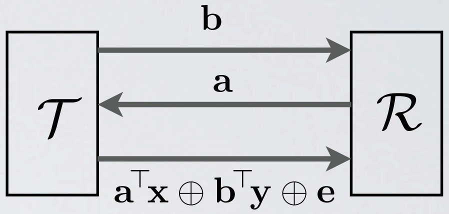

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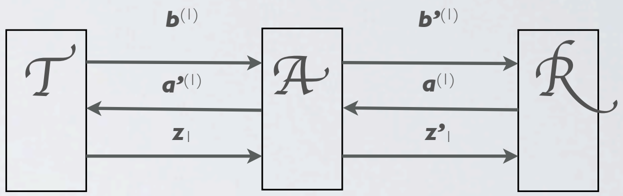

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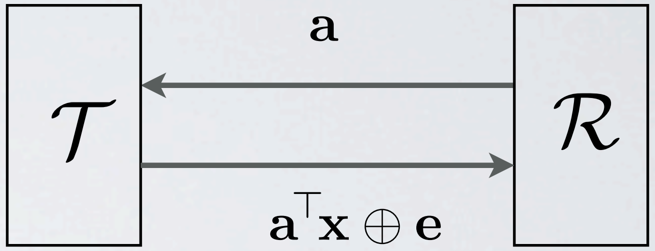
[BHN] HB^N Protocol

- Tools:
  - ⊕: [0,1] x [0,1] -> [0,1]
  - LSN <=> LPN
  - Probabilistic Verification
  - Sequence of Games

HB and HB^N

- HB is **extremely simple**:
  - Tag computes noisy parity.



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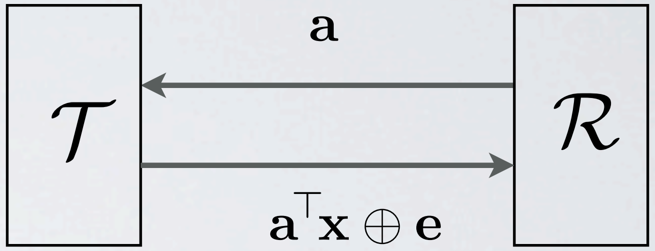

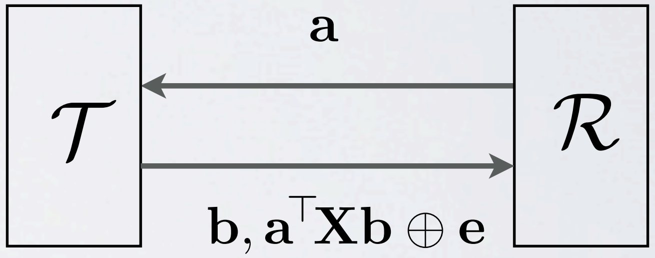
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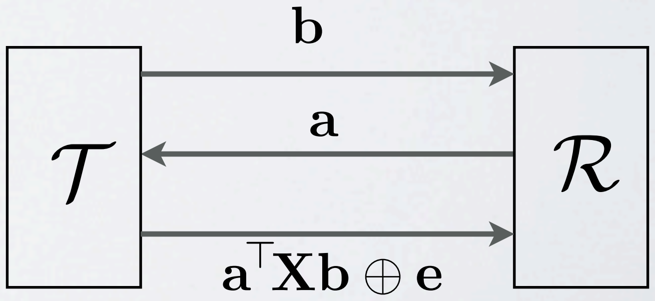

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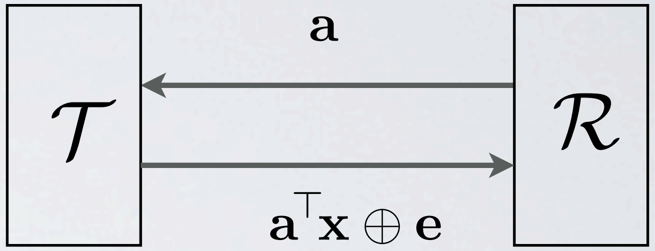

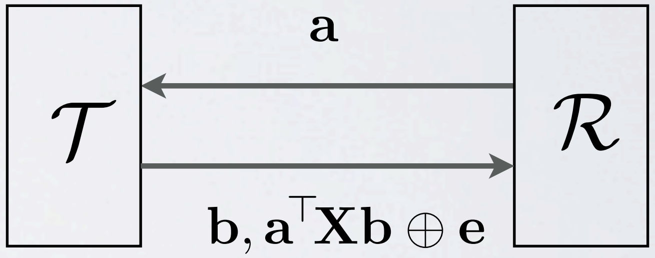
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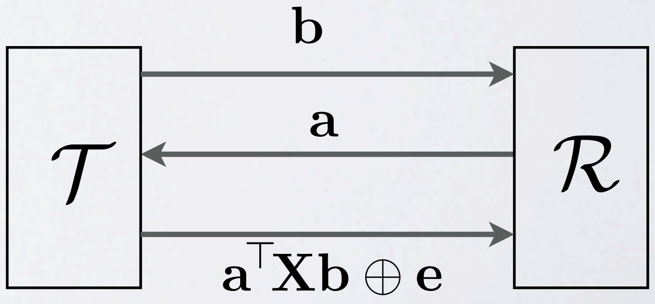

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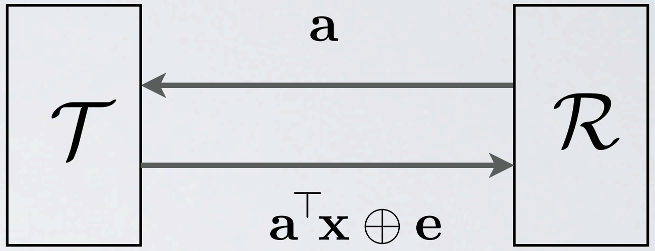
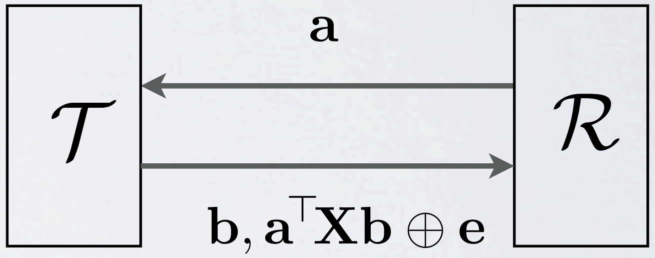
- New technique for defending against verify queries: **Probabilistic Verification.**
  - $\mathcal{R}$ computes $\mathbf{w}_i = \mathbf{a}^{\top}\mathbf{X}\mathbf{b}+\mathbf{f}_i$

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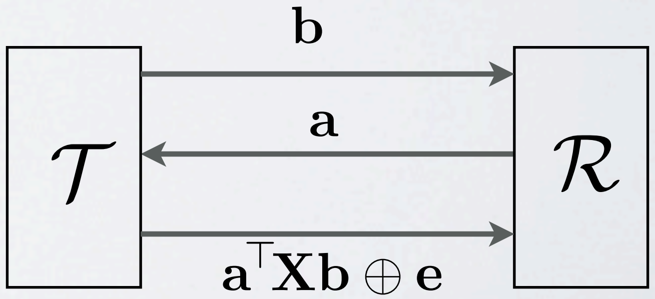

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  - $\varepsilon \oplus \rho = (1-\varepsilon)\rho + (1-\rho)\varepsilon$
  - $\oplus$ restricted to $Z_2 \times Z_2$ is equivalent to $\oplus$

Adding Noise: $\oplus$ and $\overline{\rho}$

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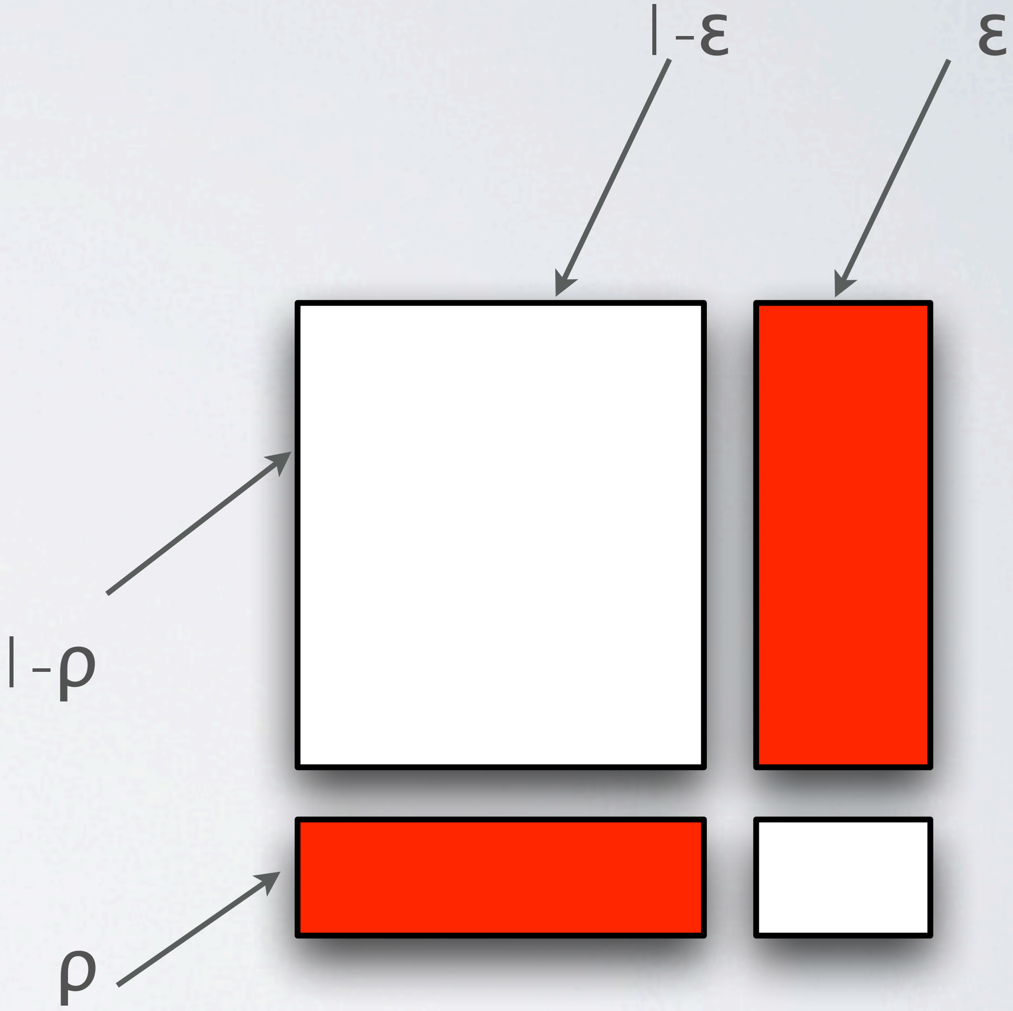
- $\Pr[\mathrm{Ber}_\varepsilon = b] = b \oplus \overline{\boldsymbol{\varepsilon}}$
- $\Pr[(\mathbf{a},b)\leftarrow\mathrm{LPN}_\varepsilon{}^{\mathbf{x}}] = 2^{-n}(\mathbf{a}^T\mathbf{x} \oplus b \oplus \overline{\boldsymbol{\varepsilon}})$
- $\Pr[(\mathbf{a},b)\leftarrow\mathrm{LSN}_{\rho,\varepsilon}{}^{\mathbf{x}}] = (b \oplus \overline{\boldsymbol{\rho}})(b \oplus \mathbf{a}^T\mathbf{x} \oplus \overline{\boldsymbol{\varepsilon}})2^{-n+1}$

$\mathrm{LPN}_{\varepsilon} \leq \mathrm{LSN}_{\rho,\varepsilon} \leq \mathrm{LPN}_{\varepsilon}$

- $\mathrm{LSN}_{\rho,\varepsilon}^{\mathbf{x}}$ is a method of producing a noisy subspace for **a**, using $\mathrm{LPN}_{\varepsilon}^{\mathbf{x}}$
  - Obtain $\breve{b}$ from Ber_{ρ}
  - Sample $\mathrm{LPN}_{\varepsilon}^{\mathbf{x}}$ until $b = \breve{b}$
- We can annihilate, **conditionally**
  - $b \leftarrow \mathrm{Ber}_{½}$ when $\mathbf{a}^{\mathsf{T}}\mathbf{x} = 1$



Game Sequence: Overall Idea

- Phase I & II keys: X_j & Y_j
  - Initially, $X_0 = Y_0$
- At each step, add random rank 1 matrix:
  - $(X,Y) \text{ -> } (X+(t+r)s^T, Y+ts^T) \text{ -> } (X, Y+rs^T)$
  - With each layer, X_j and Y_j grow further apart
  - after sufficiently many applications, $a^T X_j b^T$ is completely independent of $a^T Y_j b^T$

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  - In upcoming work [BN] obtain $\omega(n^2)$
    - via $\omega(\log n)$ rank matrix key
    - and Four Russians Matrix Multiplication trick